VIRTUAL LABORATORIES AS STRATEGY FOR TEACHING IMPROVEMENT IN MATH SCIENCES AND ENGINEERING IN BOLIVIA

LABORATÓRIOS VIRTUAIS COMO ESTRATÉGIA DE MELHORIA DO ENSINO EM CIÊNCIAS MATEMÁTICAS E ENGENHARIA NA BOLÍVIA

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Abstract. STEM education is a strategy based on four disciplines (science, technology, engineering and mathematics), integrated in an innovative interdisciplinary approach. Although, the concept of STEM education is more relevant today, the discussion of a teaching model with special attention in the four subjects aforementioned began in the early 2000s. Taking into account this context, the strategy presented in this paper has been disseminated in Bolivia's main universities for the last five years. A country that has not yet managed to associate basic disciplines such as calculus, matrix algebra, and/or differential equations to solve problems of an applicative nature, that is, to establish the link between theory and practice. To establish the connection, it is necessary to deduce differential equations associated with practical problems; solve these equations with numerical methods, appeal to the simulation concept to later introduce programming languages like Python/VPython to build virtual laboratories. The classical problem addressed for this purpose is the satellite of two degrees of freedom.

Keywords: numerical methods; programming, languages; simulation; satellite control.

Resumo. A educação STEM é uma estratégia baseada em quatro disciplinas (ciência, tecnologia, engenharia e matemática), integradas em uma abordagem interdisciplinar inovadora. Embora o conceito de educação STEM seja mais relevante hoje, a discussão de um modelo de ensino com atenção especial nestas quatro disciplinas teve início no início dos anos 2000. Levando em conta esse contexto, a estratégia apresentada neste trabalho vem sendo divulgada nas principais Universidades da Bolívia há cinco anos. Um País que ainda não conseguiu associar disciplinas básicas como cálculo, álgebra matricial e / ou equações diferenciais para resolver problemas de natureza aplicada, ou seja, estabelecer o vínculo entre teoria e prática. Para estabelecer a conexão, é necessário deduzir equações diferenciais associadas a problemas práticos; resolver essas equações com métodos numéricos, apelar ao conceito de simulação para posteriormente introduzir linguagens de programação como Python / VPython para construir laboratórios virtuais. O problema clássico endereçado para este propósito é o satélite de dois graus de liberdade.

Palavras-chave: métodos numéricos; linguagens de programação; simulação; controle de satélite.

INTRODUCTION

STEM education is a strategy that proposes teaching based on four disciplines (science, technology, engineering and mathematics), integrated in an innovative and interdisciplinary approach (Deming & Noray, 2018). The United States, the most innovative country, home to the world's top startups, has identified the need to invest in these disciplines as a priority. For these reasons and looking for a high level of competitiveness in the professional market, several universities around the world have introduced computing tools (Matlab-Simulink®, Mathematica™, Python, etc.), in their teaching programs which has helped to address a wide type of problems and increase the level of understanding not only in sciences and engineering but also in other STEM areas (Gilat, 2006), (Lopez & Sanches, 2011), (Wilson, 2015).

Some universities in Brazil, Chile, Argentina and Colombia have already adopted the STEM concept (Espinal & Segui, 2019). Contrary to this trend in South America, most universities in Bolivia do not apply it. There is a resistance to a methodological change, and to the use of technology to improve education, at the same time research works are just beginning. In this sense, this document aims to present the strategy proposed in Bolivia to improve education through the implementation of virtual laboratories. The strategy presented has been disseminated in the main Bolivian universities (Bolivia has 10 state and over 30 private universities).

Bolivian problem

For the past five years, different activities have been executed with several Bolivian universities (advanced courses, symposium and recently, master's degrees courses) (Vargas, 2016).

As a result of this interaction, it has become clear that some universities face several problems, among them, the lack of funds for the purchase of specialized software. At the same time, they have been slow in

updating their study programs and in the application of modern techniques in higher education. These and other aspects do not allow universities to be much more innovative. These problems increase due to structural failures in high school education (i.e. reduced hours in math sciences) and successive changes of methodology in teaching schools where teachers of basic school are trained (Vargas, 2018).

The strategy presented in this document was disseminated in several Bolivian universities as a proposal to improve teaching in math sciences and engineering. In the proposal, was also considered students' comments such as, what do I do with so much mathematics? Calculus I, II, III, differential equations, numerical methods? where do I apply all this? These questions are constantly being asked because math science disciplines are still administered without a link of applicative character. This work tries to establish this link as the main contribution to Bolivian universities (Vargas, 2018).

The proposal strategy

Figure 1 illustrates the chronological sequence of the strategy that starts from the basic cycle that contains disciplines such as calculus and physics, an intermediate cycle that pro-vides knowledge in numerical methods, automatic control and software languages such as Python/VPhyton. In sequence there are advanced concepts such as simulation and visualization and analysis of the obtained results.

In this sense, section II presents basic modeling concepts, section III presents a summary of the Python and Vpython tools for building graphical interfaces. Section IV presents the problem of the two degrees of freedom satellite, its mathematical modeling, simulation and graphical interface. Section V contains the corresponding control problem. Finally, some conclusions and discussions are presented in section VI.

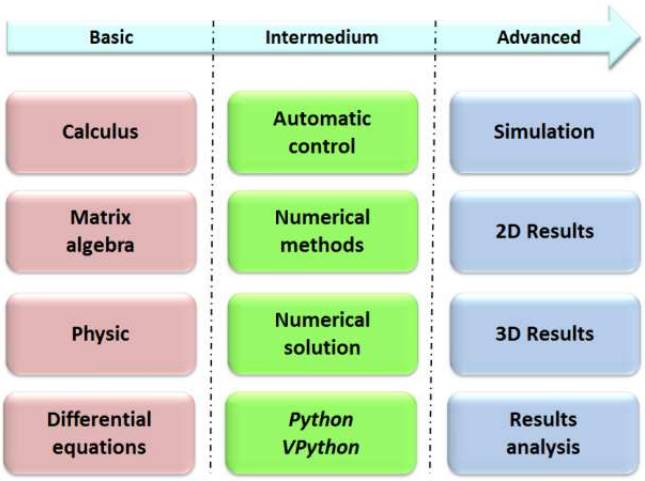


Figure 1. Chronological sequence strategy. Source: Own elaboration.

BASIC CONCEPTS

Most of the phenomena that occur in nature involve the variation of one quantity related other, leading naturally to mathematical models based on differential equations[1] (Kupferman, 2008). To obtain these equations, physical laws such as: Newton's second law, the law of conservation of energy, Kirchhoff's law, or those corresponding to a phenomenon are used. Some of these basic concepts are presented below.

Euler-Lagrange equations

A dynamic system with *n* degrees of freedom in generalized coordinates[2] $q \in \mathcal{R}^n$ and external forces $Q_\ell \in \mathcal{R}^n$ is described by the following **Euler-Lagrange** equation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}(\dot{q},q)\right) - \frac{\partial \mathcal{L}}{\partial q}(\dot{q},q) = Q_\ell \qquad (1)$$

where:

[1] They can also algebraic equations.
[2] Let the set of variables $q_1....q_4$.

$$\mathcal{L} = \mathcal{T}(\dot{q}, q) - \mathcal{V}(q) \tag{2}$$

corresponds to *Lagrange* function, $\mathcal{T}(\dot{q}, q)$ is the kinetic energy and $\mathcal{V}(q)$, is potential energy.

The external forces considered are friction, control input and the interaction between system and environment. The control vector considered is $u \in \mathcal{R}^m$. Friction forces are represented by $\frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q})$, where $\mathcal{F}(\dot{q})$ is the function of **Rayleigh.** In summary, the following external forces are considered:

$$Q_\ell = -\dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) + Q_\epsilon + u \tag{3}$$

where Q_ϵ is the signal that contains external perturbations.

Mathematical representation

Models are represented by the following differential equations:

$$\begin{aligned} \dot{x}_1 &= f_1(t, x_1, x_2, \ldots, x_n, u_1, \ldots, u_m) \\ \dot{x}_2 &= f_2(t, x_1, x_2, \ldots, x_n, u_1, \ldots, u_m) \\ &\vdots \\ \dot{x}_n &= f_n(t, x_1, x_2, \ldots, x_n, u_1, \ldots, u_m) \end{aligned} \tag{4}$$

where $\dot{x}_i$ denotes the derivative of x_i related at the time *t* while, $u_1, \ldots, u_m$ are the control variables and $x_1, \ldots, x_n$ correspond to the state variables, with $x \in \mathcal{R}^n$ and $u \in \mathcal{R}^m$. The output variables are given by:

$$\begin{aligned} y_1 &= h_1(t, x_1, x_2, \ldots, x_n) \\ y_2 &= h_2(t, x_1, x_2, \ldots, x_n) \\ &\vdots \\ y_p &= h_p(t, x_1, x_2, \ldots, x_n) \end{aligned} \tag{5}$$

where $y \in \mathcal{R}^p$. Finally, equations (4) and (5) can be written as:

$$\dot{x} = f(t, x, u) \tag{6}$$
$$y = h(t, x, u) \tag{7}$$

Equilibrium points

The equilibrium points are solutions of the equations (6) and (7) such as:

Definition 2.1 The point $x = x^*$ in the state space is an equilibrium point of $\dot{x} = f(t, x)$ if:

$$x(t_0) = x^* \Rightarrow x(t) = x^*; \quad \forall t \geq t_0 \tag{8}$$

that is, the equilibrium point is (x^*, u^*).

In simple problems (like the pendulum), the solution of equation (8) is straightforward (Khalil, 2002). However, when the degree of difficulty increases, extracting operating conditions is also more complex; an aircraft is a good example (Brian & Frank, 2003), (Vargas, Oliveira Moreira, & Paglione, 2016).

Linearization

The linearization at a point of operation is a common practice in engineering to analyze nonlinear systems. Once the linear model is obtained, its analysis is done with widely used tools (Ogata, 2011). However, the approximation around an operation point offers the local behavior of the non-linear system and not the global behavior in which we are interested. One of the most common methods to perform the linearization

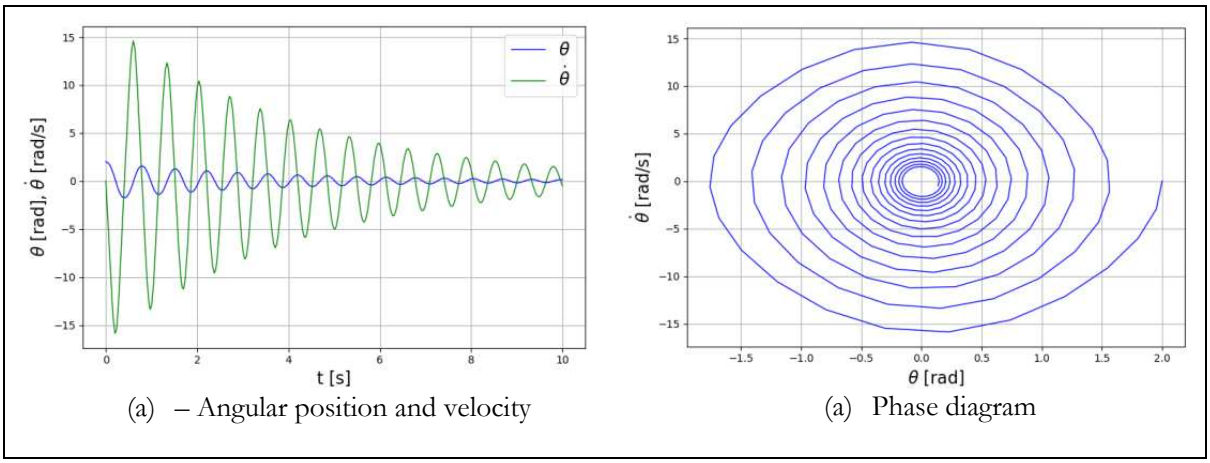
is the expansion by series of Taylor, effective for small variations around the operation point (Duke, Antoniewicz, & Krambeer, 1988), (Gangsaas, Hodgkinson, & Harden, 2008).

PYTHON AND VPYTHON LANGUAGES

Python is a high-level interpreted language with sequential commands developed at the end of the 1980s by Guido van Rossum at the National Research Institute of Mathematics and Computer Science in the Netherlands (Kusalaas, 2013). This language is widely used in large organizations due to its multiple programming paradigms. It provides standard libraries that include areas such as: chain operations, internet, web service tools, operational system interfaces and protocols. The Python interpreter is developed under an open source license, which makes it free to use and distribute, even for commercial purposes (Hetland, 2008). This last characteristic makes it ideal for its use, since it dispenses the license payment reducing costs in universities and industries. Figure 2 presents a Python code example (corresponds to the simples' pendulum problem (Vargas & Paglione, 2015)). Figures. 3(a) and 3(b) allow to verify the behavior of a simple pendulum in two dimensions.

```
from numpy import _
import numpy as np from s c ipy . i n t e g r a t e
import ode int import ma tpl o t l ib . pyplot as p l t
# Parameters
g = 9 . 8 ; L = 0 . 1 ; m = 0 . 2 ; u = 0 . 0 ; k = 0 . 1
de f eqs mov ( s tat e , t , p ) : # dy/dt = f ( t , y )
y1 , y2 = s t a t e
dydt = np . z e r o s l i k e ( s t a t e )
dydt = [ y2 ,
−(g/L)_np . s i n (y1)−(k/m)_y2 + u/(m_L__2 ) ]
r e turn dydt
# I n i t i a l Condi t ions
p = [ g , L, k , u , m] ; s t a t e = [ 2 . , 0 ]
t =np . l i n s p a c e ( 0 , 1 0 , 3 0 0 ) ; pr int ( t )
s o l = ode int ( eqs mov , s tat e , t , args=(p , ) )
```

Figure 2. Python code for pendulum model. Source: Own elaboration



(a) – Angular position and velocity (a) Phase diagram

Figure 3. Simples' pendulum behavior. Source: Own elaboration

In sequence to the development of the language Python in the spring and summer of the year 2000, with the help of David Andersen, Ruth Chabay, Ari Heitner, Ian Peters and Bruce Sherwood, David Scherer creates Visual, a module for Python that was not only relatively simple to use, but also allowed the development of objects in three dimensions. The combination of Python and Visual is called VPython which allows users to create objects such as spheres, cylinders and 3D cones and displays them in a window, which facilitates the creation of three-dimensional visualizations. This last feature has made VPython an essential tool for the creation of virtual laboratories (Villate, 2018).

The three-dimensional representation of the double pendulum is illustrated in Figure 4.

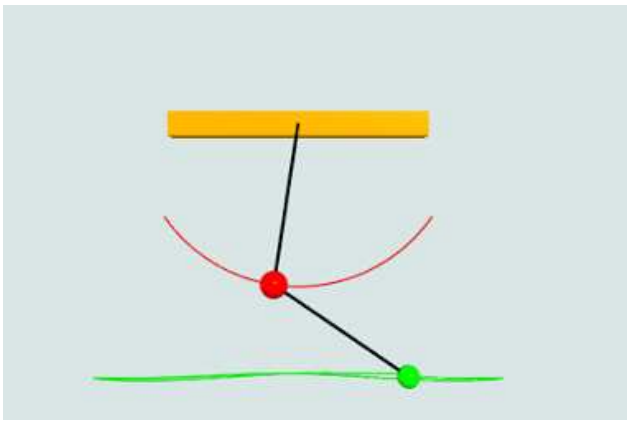


Figure 4. Virtual environment for double pendulum. Source: Own elaboration

TWO DEGREE OF FREEDOM SATELLITE PROBLEM

The orbit of the satellite of two degrees of freedom with mass *M* is illustrated in Figure 5, can be described by two non-linear and coupled equations obtained from the sum of the kinetic energy due to the tangential component $v_t = r(t)\dot{\alpha}(t)$ and radial component of speed $v_r = \dot{r}(t)$ It is assumed that the satellite has traction in these directions, thus the magnitude of the forces are F_t and F_r respectively.

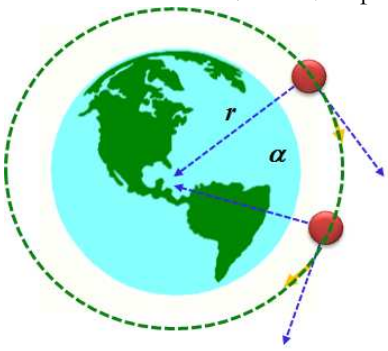


Figure 5. Virtual environment for pendulum. Source: Own elaboration

This problem is well known in the area of automatic control, but, is not studied in the Bolivia education program with multiple inputs and multiple outputs (MIMO) (Siqueira, Moreira, & Paglione, 2007), (Vargas & Irusta, 2014).

Mathematical model

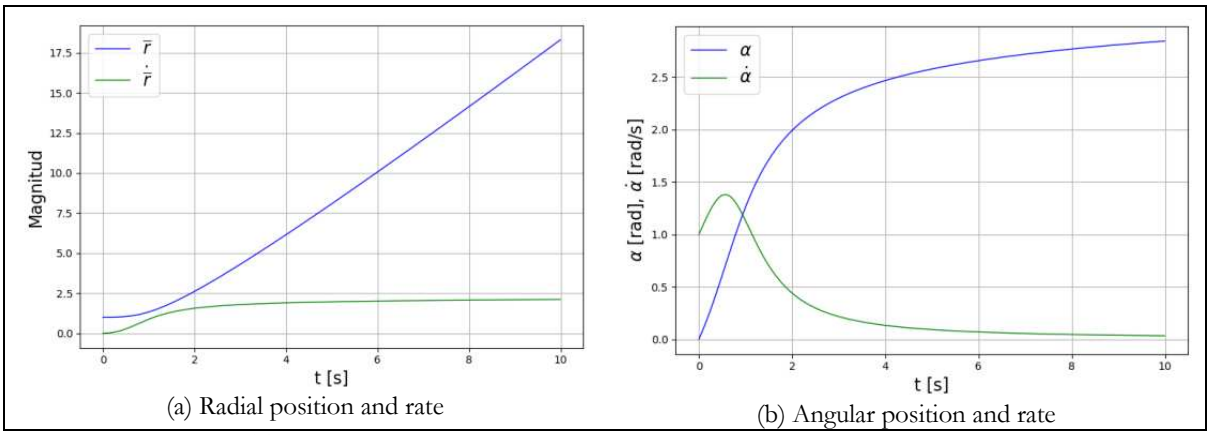
The total kinetic energy of the satellite of Fig. 5 is given by:

$$\mathcal{T} = \frac{1}{2}M[v_t^2 + v_r^2] = \frac{1}{2}M[r^2(t)\dot{\alpha}^2(t) + \dot{r}^2(t)] \quad (9)$$

The equations (9) and (2) allow obtaining the mathematical model of this problem (Wolovich, 1994).

Computational solution

Figure 6(a) illustrates the results obtained for *r(t)* and $\dot{r}(t)$. It is verified that the *r(t)* moves away from its initial condition through a ramp, for this reason the derivative $\dot{r}$ tends to be a constant.



(a) Radial position and rate (b) Angular position and rate

Figure 6. Two Degree of Freedom Satellite in two dimensions. Source: Own elaboration

Figure 6(b) illustrates the results obtained for $\alpha(t)$ and $\dot{\alpha}(t)$. It is verified that $\alpha(t)$ remains with constant behavior over time, for this reason the derivative $\dot{\alpha}(t)$ tends to zero. In practice, it is expected that the satellite will describe a trajectory with constant angular velocity $\dot{\alpha}(t)$ and with position *r(t)* constant also.

The Figure 7 illustrates the tridimensional virtual environment for the satellite of two degrees of freedom.

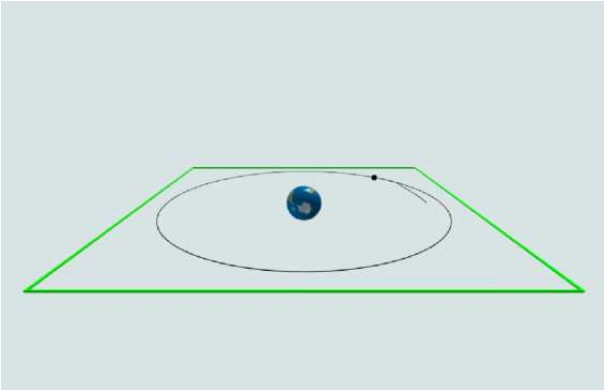


Figure 7. Virtual environment for pendulum. Source: Own elaboration

AUTOMATIC CONTROL PROBLEM

The analysis and synthesis are performed by applying theory of optimal control and the technique of Linear Quadratic Regulator (LQR), method based on the linearization of dynamic systems, represented in the time domain (Brian & Frank, 2003), (Ogata, 2011). The control structure is presented in Figure 8.

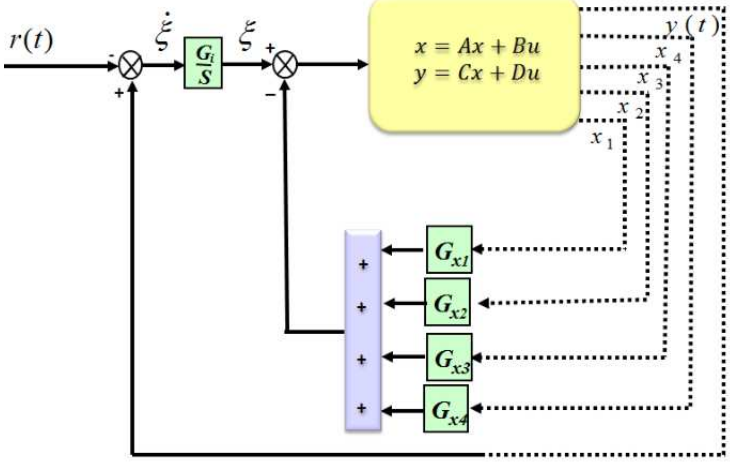


Figure 8. Generic structure control. Source: Own elaboration

Algorithm

The algorithm to find a control law u[3] is given by:

$$\begin{aligned} u &= -[G_{x1}\ G_{x2}\ G_{x3}\ G_{x4}]x + G_i\xi \\ \dot{\xi} &= r - y \end{aligned} \qquad (10\text{-}11)$$

so that the feedback control law minimizes the *performance index*:

$$J = \int_0^\infty \{x^* Q x + u^* R u\} dt \qquad (12)$$

subject to the following state equation:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad (13)$$

The gain matrix $G = [G_{x1}\ G_{x2}\ G_{x3}\ G_{x4}\ G_i]$ is obtained from the solution of **Riccati's** equation.

$$A_{aum}^T P + P A_{aum} - P B_{aum} R^{-1} B_{aum}^T P + Q = 0 \quad ()$$

where, A_{aum} and B_{aum} are the augmented matrices. The optimal gain G can be obtained quickly using Python Control Toolbox.

The code used to calculate the gains is presented in Figure 9.

```
from control import _ from sly cot import _
from control. matlab import _
import numpy as np
#Dates
Jm = 1.2; Bm = 0.1; Jl = 0.3;
Bl = 0.05; m = 1; Kr = 0.5;
l= 0.5; g = 9.8;
A = [[0, 1], [-(g_l_m) / (Jl+Jm_Kr__2),
-(Bl+Bm_Kr__2)/(Jl+Jm_Kr__2)]];
B = [[0], [1 / (Jm+(Jl/Kr__2))]];
C = [[1, 0], [0, 1]]; D = [[0], [0]];
Aaum = [[A], [[0], [0]], [B], [1]] Baum = [[B], [0]]
Q = [[20, 0, 0], [0, 20, 0], [0, 0, 20]] R = 0.01
G, S, E =lqr(Aaum,Baum, Q, R)
```

Figure 9. Generic structure control. Source: Own elaboration

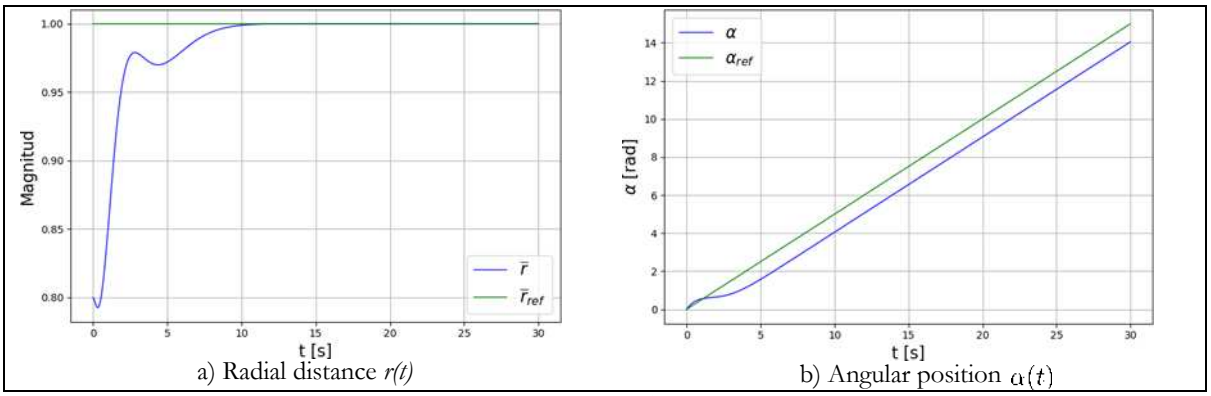
Two degree of freedom satellite control

The controlled variables correspond to the position in relation to the center of the earth and the angular position of the satellite. The desired references are the unit for |r| and a ramp 0.5t for α. Initials conditions of the integrator are $[0.8\ 0\ 0\ 1]^T$. The gain matrix *G* obtained is:

$$\begin{bmatrix} 15.0466 & 8.3576 & -3.3905 & 0.0601 & -6.0658 & 1.7907 \\ 3.5564 & 0.0601 & 11.4848 & 7.9502 & -1.7907 & -6.0658 \end{bmatrix} \qquad (15)$$

Figure 10(a) shown the result of radial component *r(t),* where is verified that after of transient, the reference is reached.

[3] In practice, the control variable *u*, it is always limited to its maximum and minimum values.



a) Radial distance *r(t)* b) Angular position $\alpha(t)$

Figure 10. Closed loop results satellite TDOF. Source: Own elaboration

Figure 10(b) illustrates the result obtained for the angular position $\alpha(t)$, which follows the specified ramp with an offset, offset is characteristic of **LQR** technique for ramp reference. To solve this problem, it is necessary to study other control strategies (Wolovich, 1994)

CONCLUSIONS AND DISCUSSIONS

The main contribution of this proposal is applicative, simple examples show the link between theory and practice, the results obtained through simulations are evaluated in two and three dimensions, which allow the extensive use of virtual laboratories. The presented strategy is used in countries with high development indices, unfortunately this does not happen in Bolivia.

The summary of the methodology that addresses the problems presented is illustrated in Figure 11.

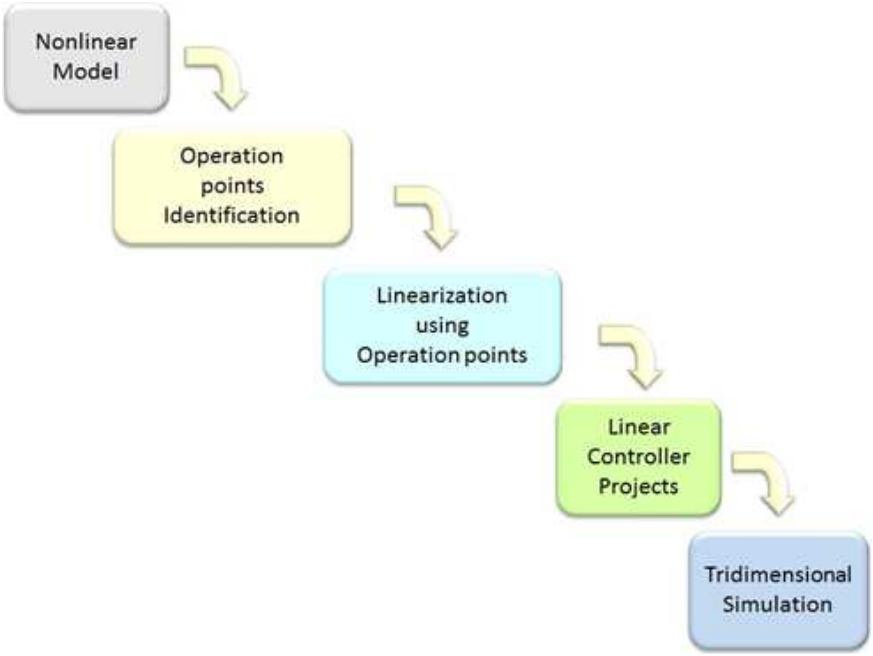


Figure 11. Summary of methodology. Source: Own elaboration

The main advantage of the presented strategy is for the country, which will benefit from a future generation with more professionals capable of working in the new industrial era. In this sense, it should be emphasized that this strategy will help in the training and motivation of the new Bolivian scientists in the short- and long-term period. This strategy is also attractive because programming languages like *Python/Vpython* are free. Let us not forget that these Virtual Laboratories can be quickly implemented on any computer and managed from a dedicated server that doesn't need the purchase of additional hardware.

The total number of attendees where the strategy was presented reached approximately 500 among professors and students. During the presentations, a questionnaire was pre-pared asking what methodology was practiced by the participating universities. Figure 12 illustrates 12 presentations with an average of 40

participants by session. The total number of participants and the percentages that correspond to the methodologies Non-applied method and Applied method.

The result shows that 75% still administer subjects such as calculus, algebra and others isolated from practical applications. In this sense, the participating students evidenced the improvement in learning disciplines in science and engineering, integrated to the concepts of modeling, programming languages, numerical methods and simulation. Following, some feedback from participating students are registered:

- *Great depth in the methodology, as rarely explained in our universities,*
- *I did not see this method before, the learning is transparent and real-world examples can be observed,*
- *The strategy clearly shows the complete cycle that involves the deduction of mathematical models until the visualization of results in two and three dimensions.*

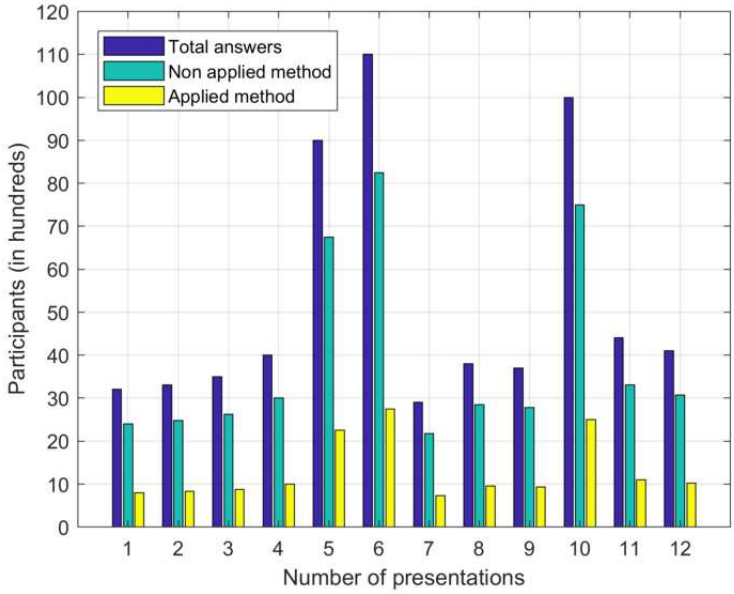
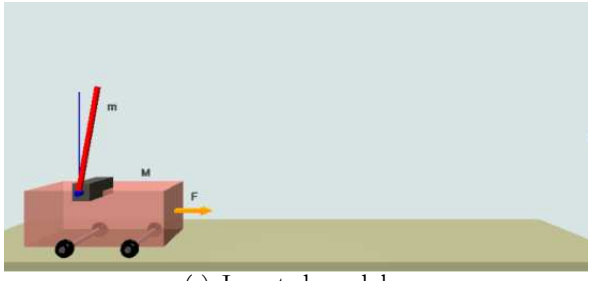
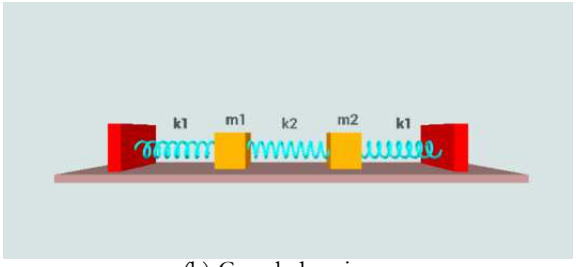


Figure 12. Number of presentations and answers of attendees. Source: Own elaboration

Furthermore, an interactive link is being developed where students can access to the presented problems and others (Alvarado, 2019). Additionally, Figures 13(a) and 13(b) show the virtual environment for inverted pendulum and coupled springs problems



(a) Inverted pendulum



(b) Coupled springs

Figure 13. Additional virtual environments. Source: Own elaboration

The participating universities were suggested to use the strategy extensively, as well as to change their science and engineering study plans. After these recommendations were made, some universities have very slowly begun to make the suggested changes. Finally, during the dissemination of the strategy, two books were published, one in Bolivia and another in Brazil (Vargas & Irusta, 2014), (Vargas & Paglione, 2015).

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