ASSESSING THE PRECISION OF GRIDDING TECHNIQUES FOR CREATING SURFACES AND CALCULATING VOLUMES THROUGH DIFFERENT HYPO-THETICAL TERRAINS

AVALIAÇÃO DA PRECISÃO DAS TÉCNICAS DE MAPEAMENTO PARA CRIAÇÃO DE SUPERFÍCIES E CÁLCULO DE VOLUMES EM DIFERENTES TERRENOS HIPOTÉTICOS

EVALUACIÓN DE LA PRECISIÓN DE LAS TÉCNICAS DE MATIZADO PARA CREAR SUPERFICIES Y CÁLCULO DE VOLÚMENES A TRAVÉS DE DIFERENTES TERRENOS HIPOTÉTICOS

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Abstract. The objective of this study is to assess the precision of various gridding methods and numerical integration approaches in accurately representing surfaces and calculating volumes between two surfaces. Three hypothetical surface categories were generated and volumes were computed using analytical integration in Python. These surfaces served as benchmarks for comparison with other surface forms and computed volumes. Various gridding methods available in Surfer software were compared, and the computed volumes were compared to those obtained from Python analytical integration. Statistical metrics such as Absolute Error, Squared Error, and Absolute Percentage Error were used to assess the precision of each approach. The results showed that methods like Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, Radial Basis Function, and Modified Shepard's Method exhibited the highest precision across all surface types. The Natural Neighbor method was inconsistent across all surface groups, whereas, the Local Polynomial Method had varying results. Polynomial Regression, Moving Average, and Data Metrics methods were excluded. The first two methods suffer from a lack of precision in representing terrain natures, whereas the third method resulted both invalid volume estimates and terrain shape representation.

Keywords: Numerical Integration; Gridding Methods; Volume Calculation, Hypothetical Surfaces, Mining Applications, Surface Topography, Surfer software.

Resumo. O objetivo deste estudo é avaliar a precisão de diversos métodos de gradeamento e abordagens de integração numérica para representar superfícies com precisão e calcular volumes entre duas superfícies. Três categorias de superfícies hipotéticas foram geradas, e os volumes foram calculados utilizando integração analítica em Python. Essas superfícies serviram como referência para comparação com outras formas de superfície e volumes calculados. Vários métodos de gradeamento disponíveis no software *Surfer* foram comparados, e os volumes calculados foram analisados em relação aos obtidos por integração analítica em Python. Métricas estatísticas como erro absoluto, erro quadrático médio e erro percentual absoluto foram utilizadas para avaliar a precisão de cada abordagem. Os resultados mostraram que métodos como *Kriging*, inverso da distância elevada a uma potência, triangulação com interpolação linear, curvatura mínima, vizinho mais próximo, função de base radial e o método de Shepard modificado apresentaram maior precisão em todos os tipos de superfícies. O método de vizinho natural apresentou inconsistência em todas as categorias de superfícies, enquanto o método polinomial local teve resultados variáveis. Métodos como regressão polinomial, média móvel e métricas baseadas em dados foram excluídos do estudo devido à baixa precisão na representação do terreno e à geração de estimativas de volume e formas de terreno inválidas.

Palavras-chave: Integração numérica, Métodos de gradeamento, Cálculo de volume, Superfícies hipotéticas, Aplicações em mineração, Topografia de superfícies, Software Surfer.

Resumen. El objetivo de este estudio es evaluar la precisión de varios métodos de cuadrícula y enfoques de integración numérica para representar superficies con precisión y calcular volúmenes entre dos superficies. Se generaron tres categorías de superficie hipotéticas y se calcularon volúmenes utilizando la integración analítica en Python. Estas superficies sirvieron como puntos de referencia para la comparación con otras formas de superficie y volúmenes calculados. Se compararon varios métodos de cuadrícula disponibles en el software Surfer y los volúmenes calculados se compararon con los obtenidos a partir de la integración analítica de Python. Se utilizaron métricas estadísticas como el error absoluto, el error cuadrático y el error porcentual absoluto para evaluar la precisión de cada enfoque. Los resultados mostraron que métodos como Kriging, la distancia inversa a una potencia, la triangulación con interpolación lineal, la curvatura mínima, el vecino más cercano, la función de base radial y el método de Shepard modificado exhibieron la mayor precisión en todos los tipos de superficie. El método del vecino natural fue inconsistente en todos los grupos de superficies, mientras que el método polinomial local tuvo resultados variables. Se excluyeron los métodos de regresión polinomial, promedio móvil y métricas de datos. Los dos primeros métodos adolecen de una falta de precisión a la hora de representar la naturaleza del terreno, mientras que el tercer método dio como resultado estimaciones de volumen y representaciones de la forma del terreno no válidas.

Palabras-clave: Integración numérica; Métodos de cuadrícula; Cálculo de volumen, Superficies hipotéticas, Aplicaciones mineras, Topografía de superficies, Software Surfer.

1. INTRODUCTION

Precise volume determination is essential in many domains such as environmental sciences, resource estimation, mining, and land surveys. It influences the process of making decisions, evaluating the amount of ore available, implementing mining techniques that can be sustained over time, carrying out excavation and filling activities, determining the feasibility of mining operations, and studying the environmental impact. This impact includes the erosion of soil, the deposition of sediment, and the effects on the evolution of natural landscapes (Gerassis et al., 2021; Tubis et al., 2020; Suh et al., 2017; Jamshidi et al., 2024).

In the realm of contemporary geospatial technology, the utilization of instruments such as unmanned aerial vehicles (UAVs), Light Detection and Ranging (LiDAR), and digital photogrammetry has brought about a significant transformation in volume calculation. This is achieved by offering high-resolution data and facilitating quicker and more precise measurements. These technologies have greatly enhanced the efficiency and precision of volume computations in comparison to conventional methods (Lee & Lee, 2022; Li & Heap, 2011).

Advancements in contour mapping, volume calculation, and assessment of natural resource reserves depend on scientifically proven and established procedures. Interpolation is the technique of estimating values in a spatial distribution without using direct measurements. This is done using different gridding methods. Similarly, volume calculations rely on proven numerical integration techniques (Karim & Howladar, 2022; Yakar & Yilmaz, 2008; Yakar et al., 2014; Yilmaz, 2010; Kumar & Sinha, 2018).

Multiple gridding techniques have been extensively employed in multiple studies across various disciplines, showcasing their efficacy and adaptability. These techniques are necessary for generating precise digital terrain models (DTMs) and conducting spatial interpolations, which are vital for applications in environmental sciences, mining, civil engineering, and other fields. An example of the use of LiDAR technology in mining excavation is its ap-

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plication for volume calculation. This technology has been recognized for its precise and efficient ability to create detailed 3D models of the terrain (Septarini, 2013; Jamshidi & Baghdadi, 2018). Furthermore, the application of geospatial data for measuring the volume of material in dredging operations showcases the dependability of gridding techniques in managing extensive environmental data (Ekun et al., 2016).

In the mining industry, a comparative investigation of several interpolation methods using multiple software packages for volume calculation of irregular objects demonstrated the need of selecting proper interpolation methods to achieve accurate volume predictions (Mohamed & Mostafa, 2019). Furthermore, volumetric assessments of stockpiles using UAV photogrammetry and total station data demonstrated the importance of precise gridding techniques in surveying and geoinformatics (Ekpa et al., 2009). A notable study conducted a comparison between geological sections and structural maps to calculate reserves of cement raw materials.

This study emphasized the need of selecting an appropriate gridding technique to achieve precise volume estimation in intricate geological formations (Bralić & Malvić, 2022). Furthermore, an extensive comparison evaluated the performance of twelve different interpolation methods using Surfer software, providing a detailed assessment of the accuracy and efficiency of various gridding methods, offering valuable guidance for selecting the appropriate method based on the specific requirements of different applications (Yang et al., 2004; Pike, 1998).

Despite these advances, the fundamental principles of gridding and numerical integration remain important. Interpolation, the process of estimating values within a spatial distribution without taking direct measurements, is strongly reliant on known gridding techniques. Furthermore, volume calculations frequently rely on established numerical integration techniques such as the Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule. Previous research has mostly focused on the applicability and performance of gridding methods across multiple fields, but it has not adequately investigated the differences between different numerical integration techniques employed in volume calculations.

They also do not provide a detailed comparison of these integration strategies across various gridding techniques. This study seeks to address this gap by carefully comparing the performance of various numerical integration approaches across several gridding methods. This study aims to find the most acceptable methods for interpolation and the most accurate numerical integration approaches for volume calculations, offering researchers and practitioners with vital insights on how to choose the best methods for their specific applications.

The primary objective of this work is to assess the efficiency of various gridding methods in conjunction with numerical integration approaches for accurately determining the volumes of intricate surfaces.

2. MATERIALS AND METHODS

The methodology for this study is scientifically organized into some critical sections to confirm a detailed analysis of surface volume calculations. These sections are: Surface Creation and Analytical Volume Calculation, which describes the Hypothetical surfaces and determines their volumes using analytical methods; Exporting Surface Coordinates for Surfer, delegating the procedure of transferring surface data to the Surfer software; Gridding Methods in Surfer, exploring the various gridding methods employed to build the surface data; Numerical Integration Methods in Surfer, describing the numerical techniques used to evaluate volumes; Comparison of Volumes, comparing the analytical and numerical volumes; and Statistical Analysis, assessing the accuracy of the different methods using statistical parame-

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ters. Each section plays a vital role in confirming the reliability and accuracy of the volume calculations, thereby setting a solid basis for the study results.

2.1. Surface Creation and Analytical Volume Calculation

The main objective of this research is to assess the precision of several numerical integration techniques and gridding methods for the purpose of determining the volumes between diverse surface types. The research starts with the construction of three Hypothetical surfaces, with distinct mathematical equations defining the upper and lower surfaces of each group. The regularity of these surfaces is a deciding factor in their selection for analytical integration-based volume computations. The three groups of surfaces are:

Group 1: Trigonometric Surfaces:

Upper Surface equation Z_top=sin(x) cos (y)+5 (1)
Lower Surface equation Z_lower=cos (x) sin (y) (2)

Group 2: Polynomial Surfaces:

$$Z_{top} = x^2 + y^2 + \sin(x + y) + 5$$

Upper Surface equation (3)

$$Z_{lower} = x^2 - y^2 + \cos(x - y)$$

Lower Surface equation (4)

Group 3: Exponential Trigonometric Surfaces:

$$Z_{top} = e^{0.1x}\sin(y) + 10$$

Upper Surface equation (5)

$$Z_{lower} = e^{-0.1y}\cos(x)$$

Lower Surface equation (6)

The surfaces were created and examined using Python on the Google Colab platform. The code comprised instructions for generating the surfaces, computing the analytical volumes, and exporting the surface data as XYZ files.

Because of the surfaces' regularity and well-defined mathematical representations, the volumes trapped between them could be computed with great precision using analytical integration. The surface data were then exported to XYZ files using the same Python script for additional investigation.

The classification of topographic features into various geomorphological units, such as plains, hills, and mountains, is crucial for comprehending the terrain's properties in mining, environmental, or civic applications (Sarwal et al., 2003; Iwahashi & Pike, 2007; Arif et al., 2024). The choice of Trigonometric Surfaces, Polynomial Surfaces, and Exponential Trigonometric Surfaces was designed to encompass a diverse grouping of natural topographical shapes encountered in many practical applications. Trigonometric surfaces depict tough terrains characterized by consecutive peaks and basins, resembling mountainous areas. They serve as a valuable tool for examining hilly and difficult terrains. Conversely, polynomial surfaces depict smooth and slightly undulating terrains, such as plains and plateaus, which are valuable for analyzing agricultural lands and flat areas with gradual slopes. Exponential Trigonometric surfaces represent terrains with steep mountains and deep valleys, they exhibit a

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resemblance to a flood plain and/or drainage valley in their physical structure, making them valuable for studying steep mountainous areas and rough regions.

Therefore, using these surfaces provides a thorough depiction of various natural topographies. This enables the assessment of the precision and efficiency of numerical integration techniques and gridding methods in computing volumes enclosed by intricate surfaces. It aids in offering more accurate and thorough suggestions for researchers and professionals involved in the fields of civil engineering, mining, and environmental engineering.

2.2. Exporting Surface Coordinates for Surfer

The surface coordinates were produced as xyz files, enabling their entry into the Surfer software for additional analysis. This step is essential because it allows for the comparison of analytical volumes with those calculated using different gridding methods provided in Surfer.

2.3. Gridding Methods in Surfer

The Surfer software was used to generate grids for the surfaces, with twelve alternative gridding approaches. These methods include the following:

Kriging

Kriging is a geostatistical interpolation method that was initially created by Krige & Kleingeld, (2005). The method is commonly employed because to its capacity to integrate the spatial correlation structure of the data using the variogram model, resulting in precise and unbiased calculations. Its application is especially valuable in industries like mining, hydrology, and environmental research due to its proficiency in managing data that is not evenly distributed and offering an assessment of the margin of error in estimations (Belkhiri et al., 2017).

The kriging estimator Z˜(x0) for a location x0 is defined as (Webster & Oliver, 2007):

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i) \tag{7}$$

Where: λi are the kriging weights, Z(xi) are the observed data values at locations xi. The weights λi are obtained by solving the kriging system of equations, ensuring the unbiasedness of the estimator and minimizing the estimation variance.

Inverse Distance to a Power

Inverse Distance to a Power is a method of interpolation that involves calculating a weighted average. It is commonly used in the fields of geostatistics and spatial analysis. This approach can serve as both a precise and a smoothing interpolator, offering versatility in dealing with different types of spatial data (Shukla et al., 2019).

In this method, the estimated value Z˜(x0) at a given grid node x0 is calculated as a weighted average of nearby observations (Ozelkan et al., 2016):

$$\hat{Z}(x_0) = \frac{\sum_{i=1}^{n} w_i Z(x_i)}{\sum_{i=1}^{n} w_i} \tag{8}$$

Where wi are the weights assigned to each observation Z(xi). The weights wi are determined based on the inverse of the distance between the grid node and the observation points, raised to a power p:

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$$w_i = \frac{1}{d(x_0, x_i)^p} \tag{9}$$

Here, d(x_0,x_i) is the distance between the grid node x0 and the observation point xi, and p is the power parameter that controls the rate of decay of the weights with distance.

2.4. Triangulation with Linear Interpolation

Triangulation with Linear Interpolation is a technique employed in GIS, computer graphics, and terrain modeling to generate smooth surfaces from sparse data points. The process entails constructing a triangulated irregular network (TIN) composed of adjacent triangles that do not overlap. This is achieved by use Delaunay triangulation to optimize the smallest angles and prevent the formation of elongated triangles. Subsequently, linear interpolation is employed to approximate values at each given position within each triangle (Gonet & Gonet, 2017).

Minimum Curvature

The Minimum Curvature Gridding Method, commonly referred to as "bicubic spline" interpolation, is a method employed to generate smooth surfaces from data points that are not evenly distributed. The objective is to reduce the overall curvature, leading to a visually appealing and seamless depiction of the data. This approach effectively solves the biharmonic problem by guaranteeing the continuity of both first and second derivatives. The process entails making an initial estimation of the grid, making iterative modifications, and continuing this process until the change in the surface reaches a predetermined tolerance level (Bronowicka-Mielniczuk et al., 2019).

Natural Neighbor

Natural Neighbor Interpolation is a technique that enhances the interpolation of data points that are not evenly distributed, while maintaining local features and preventing distortions. The process entails creating a Voronoi diagram, determining the natural neighbors for a certain interpolation point, and computing weights based on the overlapping area between the new and old Voronoi cells. The weight assigned to each neighbor is proportional to the area of the new cell (Yanalak, 2003).

Nearest Neighbor

The Nearest Neighbor Interpolation method determines the closest data point based on Euclidean distance and assigns the value of that data point to the interpolation point (Long Nguyen et al., 2020).

Local Polynomial

Local Polynomial Interpolation is a mathematical method that entails constructing a polynomial surface to approximate data points within a particular vicinity. This approach effectively captures localized fluctuations and generates interpolated surfaces that are smooth (Schaum, 2008).

Radial Basis Function

The Radial Basis Function (RBF) utilizes interpolation through the use of radial basis functions, which are a specific sort of function that solely rely on the distance from a central point (Celant & Broniatowski, 2016).

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Polynomial Regression

Polynomial regression Interpolation is a technique employed to construct a polynomial equation that accurately represents a given set of data points, enabling the prediction of values at regions where no samples were taken. This technique is advantageous for catching the fundamental patterns in the data (Conn & Scheinberg, 2008).

2.5. Modified Shepard's Method

The Modified Shepard's Method employs an inverse distance weighted least squares technique. The Modified Shepard's Method is comparable to the Inverse Distance to a Power interpolator, but the incorporation of local least squares helps to eliminate or minimize the "bull's-eye" pattern observed in the resulting contours. The Modified Shepard's Method can function as either a precise or a smoothing interpolator (Malvić, 2020).

Data metrics Method

Data metrics gridding method generates grids of information at each individual node, presenting data points used, standard deviation, variance, coefficient of variation, and median absolute deviation. These metrics are essential for statistical analysis and can be utilized to ascertain new sampling locations and generate a contour map indicating the proximity to the nearest data point.

Moving Average Method

The Moving Average gridding method calculates the values of grid nodes by taking the average of the data points within the search ellipse. While it is possible to include breakline data, it is not advisable for small or moderate-sized data sets.

2.6. Numerical Integration Methods in Surfer

The Surfer software employs three numerical integration methods to calculate volumes:

Trapezoidal Rule

The Trapezoidal Rule is a computational technique employed to estimate the definite integral of a function. Due to its uncomplicated implementation and relatively accurate results for various sorts of functions, it is considered one of the easiest and most frequently employed methods. The fundamental concept behind the Trapezoidal Rule involves partitioning the region beneath the curve into a sequence of trapezoids, determining the area of each trapezoid, and subsequently aggregating these areas to estimate the overall integral (Lee, 2019). The equation for the Trapezoidal Rule is given by:

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right] \quad (10)$$

where: [a, b] is the interval over which the integration is performed. n is the number of subintervals. xi are the points dividing the interval [a, b] into n subintervals. The interval is divided into n equal parts, and the function values at these points are used to calculate the area of the trapezoids.

Simpson's Rule

Simpson's Rule is a more accurate method of numerical integration than the Trapezoidal Rule. Instead of trapezoids, it divides the area under the curve into parabolic pieces to approximate a function's integral. This approach provides a better level of precision, particularly

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for functions that are well approximated by parabolas across each subinterval (Lee, 2019). The equation for Simpson's Rule is given by:

$$\int_a^b f(x)dx \approx \frac{b-a}{6n}\left[f(a) + 4\sum_{i=1,3,5,...}^{n-1} f(x_i) + 2\sum_{i=2,4,6,...}^{n-2} f(x_i) + f(b)\right]$$

(11)

where: [a, b] is the interval over which the integration is performed. n is the number of subintervals, which must be an even number. xi are the points dividing the interval [a, b] into n subintervals.

Simpson's 3/8 Rule

The Simpson's 3/8 Rule is a method that expands upon Simpson's Rule for the purpose of numerical integration. It is employed to estimate the exact integral of a function by partitioning the region beneath the curve into cubic polynomials. This approach offers a superior level of precision in comparison to the usual Simpson's Rule, particularly for functions that may be well-approximated by cubic polynomials inside each subinterval (Lee, 2019). The equation for Simpson's 3/8 Rule is given by:

$$\int_a^b f(x)dx \approx \frac{3(b-a)}{8n}\left[f(a) + 3\sum_{i=1,2,4,5,7,8,...}^{n-1} f(x_i) + 2\sum_{i=3,6,9,...}^{n-3} f(x_i) + f(b)\right]$$

(12)

where: [a, b] is the interval over which the integration is performed. n is the number of subintervals, which must be a multiple of 3. xi are the points dividing the interval [a, b] into n subintervals.

The volumes between the upper and lower surfaces of each of the three surface groups were computed analytically using Python. These volumes were used as the benchmark for comparing the volumes calculated by Surfer using various gridding methods and numerical integration techniques. The volumes obtained by the three numerical integration techniques in Surfer (Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule) were compared to the volumes produced analytically, which served as the benchmark for accuracy.

2.7. Statistical Analysis

Statistical analysis plays a vital role in assessing the effectiveness of various computational methods and guaranteeing the precision and reliability of the results. This study utilized statistical metrics, including Absolute Error (AE), Squared Error (SE), and Absolute Percentage Error (APE), to compare the volumes obtained by Surfer's numerical integration methods with the volumes estimated analytically. These metrics offer valuable information about the volume and reliability of the errors linked to each gridding and integration approach, enabling a thorough evaluation of their performance. The study assists in identifying the most precise methods and those that may cause substantial errors, hence directing the selection of suitable strategies for surface volume computations in different applications.

Absolute Error (AE)

The Absolute Percentage Error calculates the error by normalizing it with respect to the analytical volume and expressing it as a percentage (Spray, 1986). It is characterized or described as:

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$$AE = |V_{numerical} - V_{analytical}| \tag{13}$$

Squared Error (SE)

The Squared Error is a metric that quantifies the variance of the errors by squaring the error, thereby emphasizing the larger differences. It is defined as:

$$SE = (V_{numerical} - V_{analytical})^2 \tag{14}$$

Absolute Percentage Error (APE)

The error is normalized relative to the analytical volume by the Absolute Percentage Error, which is expressed as a percentage (Hyndman & Koehler, 2006). It is defined as:

$$E = (V_{numerical} - V_{analytical})^2 \tag{15}$$

These statistical measurements offer a thorough assessment of the effectiveness of the numerical integration methods, emphasizing the precision and possible inconsistencies in the volume computations.

3. RESULTS

This section provides the results obtained by analyzing the distinct surface groups and comparing the volumes derived using different gridding and numerical integration methods. The subject matter is partitioned into three distinct sections:

- Results and Interpretation of Trigonometric Surfaces
- Results and Interpretation of Polynomial Surfaces
- Results and Interpretation of Exponential Trigonometric Surfaces

Each part includes detailed results, statistical analysis, and interpretations to evaluate the accuracy and reliability of the methods used.

3.1. Results and Interpretation of Trigonometric Surfaces

The first set of surfaces was generated using trigonometric functions defined by equations (1) and (2) through Python programming. The computed volume enclosed by these surfaces is 500 units. Figure 1 depicts the surfaces, highlighting the shape and spatial correlation between the upper and bottom surfaces.

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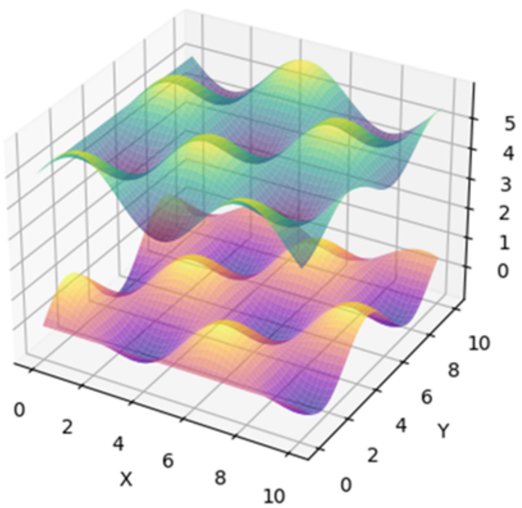

Figure 1. Trigonometric surfaces were created using Python, based on equations (1) and (2), and have a volume of 500 cubic units.

In order to assess the precision of numerical integration techniques, the volumes enclosed by these surfaces were also computed using Surfer software. The surfer uses twelve distinct gridding techniques to generate organized grids based on the surface data. The volumes were computed for each gridding approach using three numerical integration techniques: The Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule. Thus, for every gridding technique, there are three estimations of volume.

Table 1 displays the outcomes of the numerical volume calculations for each of the twelve gridding approaches, utilizing the three numerical integration techniques. The table also contains the statistical parameters—Absolute Error (AE), Squared Error (SE), and Absolute Percentage Error (APE)—that were utilized to evaluate the accuracy of the numerical volumes by comparing them with the analytically calculated volume.

Table 1. Numerical volumes and statistical parameters for trigonometric surfaces using twelve gridding methods and three numerical integration techniques, compared to the analytical volume of 500 cubic units.

Gridding Method	Numerical Integration Method	Numerical Volume	Absolute Error (AE):	Squared Error (SE)	Absolute Percentage Error (APE)
Kriging	1*	499.99999990978	0.00000009022	0.00000000000001	0.000000018044
	2**	499.99999990279	0.00000009720998	0.00000000000001	0.000000019442
	3***	499.99999990927	0.00000009073	0.00000000000001	0.000000018146
Inverse Distance to A Power	1	500	0.0	0.0	0.0
	2	500	0.0	0.0	0.0
	3	500	0.0	0.0	0.0
Triangulation with Linear Interpolation	1	500	0.0	0.0	0.0
	2	500	0.0	0.0	0.0
	3	500	0.0	0.0	0.0
Minimum Curvature	1	500	0.0	0.0	0.0
	2	500	0.0	0.0	0.0
	3	500	0.0	0.0	0.0
Natural Neighbor	1	489.95000510152	10.04999489848	101.002397459474	2.009998979696

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	2	491.61791880647	8.38208119352998	70.259285134929	1.676416238706
	3	492.45293847567	7.54706152432999	56.9581376520221	1.509412304866
Nearest Neighbor	1	500	0.0	0.0	0.0
	2	500	0.0	0.0	0.0
	3	500	0.0	0.0	0.0
Local Polynomial	1	499.99779763905	0.00220236094998	0.00000485039375	0.00044047219
	2	499.99511366893	0.00488633107	0.00002387623133	0.000977266214
	3	499.99923702114	0.00076297886	0.00000058213674	0.000152595772
Radial Basis Function	1	500.00000004354	0.00000004354001	0.0	0.000000008708
	2	500.00000003916	0.00000003916	0.0	0.000000007832
	3	500.00000004052	0.00000004051998	0.0	0.000000008104
Polynomial Regression	1	500	0.0	0.0	0.0
	2	500	0.0	0.0	0.0
	3	500	0.0	0.0	0.0
Modified Shepard's Method	1	500	0.0	0.0	0.0
	2	500	0.0	0.0	0.0
	3	500	0.0	0.0	0.0
Data Metrics	1	0	500.0	250000.0	100.0
	2	0	500.0	250000.0	100.0
	3	0	500.0	250000.0	100.0
Moving Average	1	500.00000004354	0.00000004354001	0.0	0.000000008708
	2	500.00000003916	0.00000003916	0.0	0.000000007832
	3	500.00000004052	0.00000004051998	0.0	0.000000008104

*Trapezoidal Rule **Simpson's Rule ***Simpson's 3/8 Rule

When comparing the volumes computed using the twelve gridding methods to the analytically determined volume of 500 cubic units, several observations can be made about the precision and accuracy of these methods. The volume calculations were performed using three numerical integration techniques: Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule.

There are minimum Absolute Percentage Errors (APE), Squared Errors (SE), and Absolute Errors (AE), and the volumes computed by the three Kriging numerical integration methods are very near to the analytical volume. This proves that Kriging works wonderfully and accurately with this dataset. Likewise, when it comes to volume calculations, Inverse Distance to A Power is quite reliable, since it agrees exactly with the analytical volume using all three numerical integration methods (AE, SE, and APE all equal to zero). In addition to perfectly aligning with the analytical volume, triangulation with linear interpolation shows no mistakes when using any integration method. When it comes to numerical integration methods, Minimum Curvature works wonders, producing error-free volumes.

Natural Neighbor, on the other hand, reveals substantial differences. It appears that Natural Neighbor is not as trustworthy on this dataset because the computed volumes differ significantly from the analytical volumes, leading to higher AE, SE, and APE values. However, just like the top-performing approaches, Nearest Neighbor demonstrates flawless accuracy with zero mistakes.

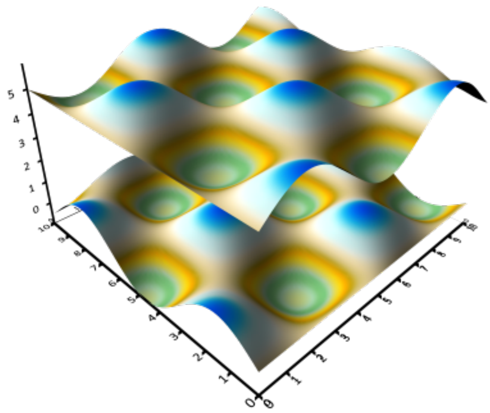
While AE and SE show the most noticeable discrepancies, Local Polynomial is often accurate. Across numerical approaches, the overall performance is solid, and these are small. Radial Basis Function equals or exceeds Kriging and Inverse Distance to A Power in terms of precision, showing essentially no AE, SE, or APE.

With no outliers in the analytical volume, Polynomial Regression proves to be absolutely accurate and dependable. In a similar vein, the analytical volume and Modified Shepard's Method match exactly, demonstrating excellent accuracy. Data Metrics, on the other hand, demonstrates terrible performance, with significant mistakes observed across all integration methods. It is clear that Data Metrics is not a good fit for this dataset because the computed volumes are drastically off. When it comes to volume calculations, Moving Average shows great accuracy with minimum errors.

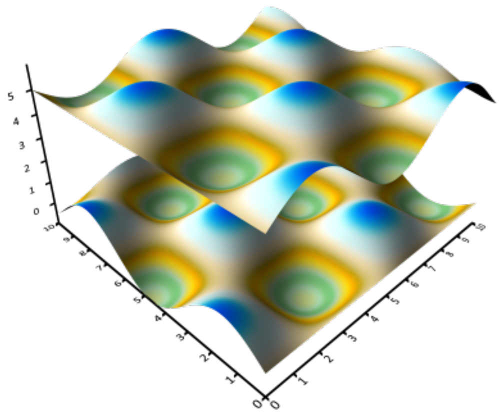
In general, techniques such as Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, Radial Basis Function, Polynomial Regression, and Modified Shepard's Method demonstrate significant precision and dependability. However, there are significant differences between Natural Neighbor and Data Metrics, indicating that they may not be suitable for analyzing this dataset. The Moving Average, although not flawless, yet demonstrates sufficient performance to be deemed credible.

To further verify the precision and visual depiction of the trigonometric surfaces generated, the Surfer software was also utilized to render the surfaces. This entailed generating grid files for each of the twelve distinct gridding techniques. The grid files were utilized in Surfer to construct visual representations of the surfaces, enabling a direct comparison with the surfaces generated by Python. The aim was to evaluate the accuracy of various gridding techniques in Surfer in reproducing the precise forms of surfaces as determined by mathematical equation.

The following are the 3D surface plots for the twelve distinct gridding methods created by Surfer for trigonometric surfaces data. Figure 2 depicts the 3D plots for each gridding method.



(**a**) Kriging



(**b**) Inverse Distance to A Power

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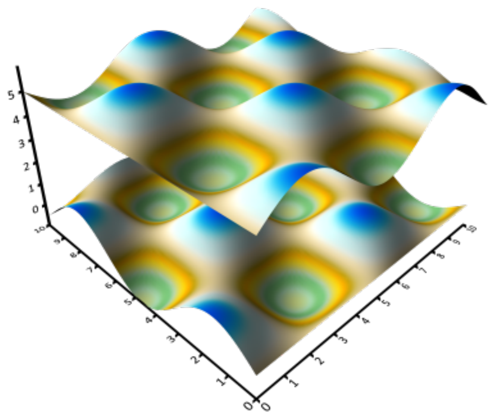
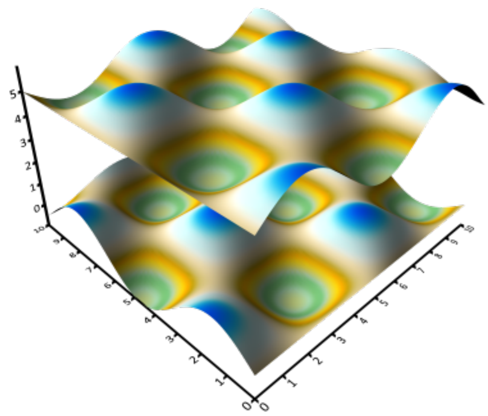
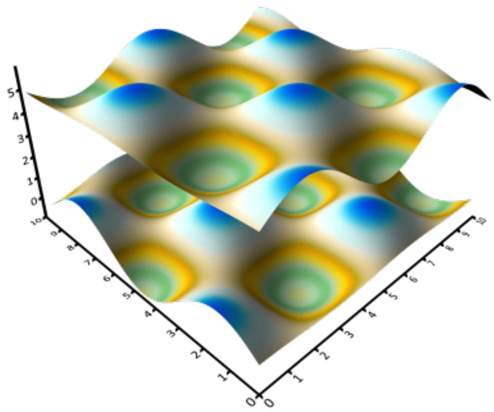


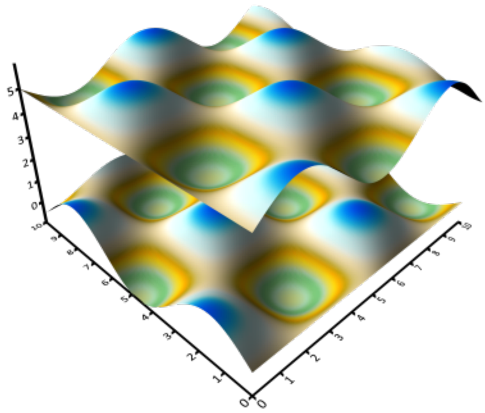
(**c**) Triangulation with Linear Interpolation



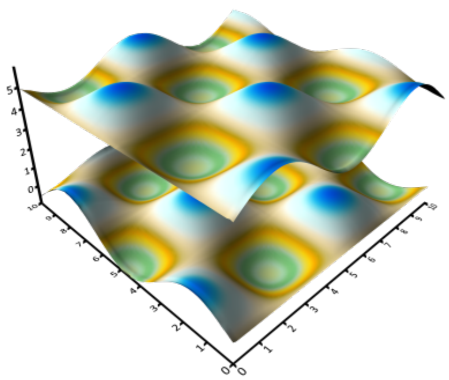
(**d**) Minimum Curvature



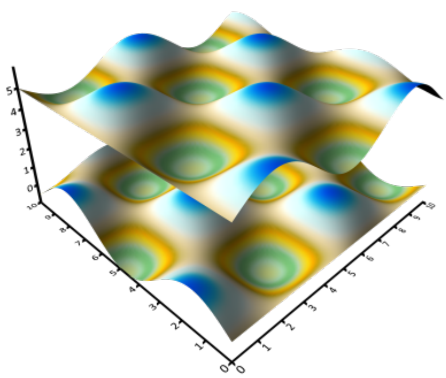
(**e**) Natural Neighbor



(**f**) Nearest Neighbor



(**g**) Local Polynomial



(**h**) Radial Basis Function

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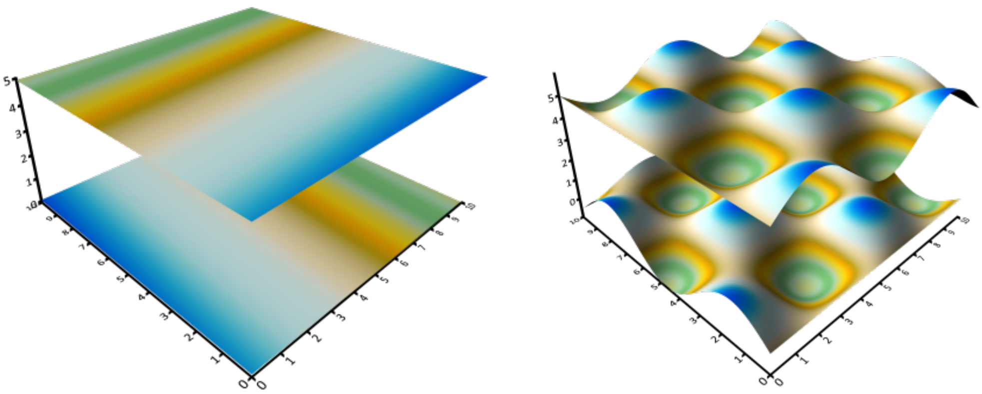
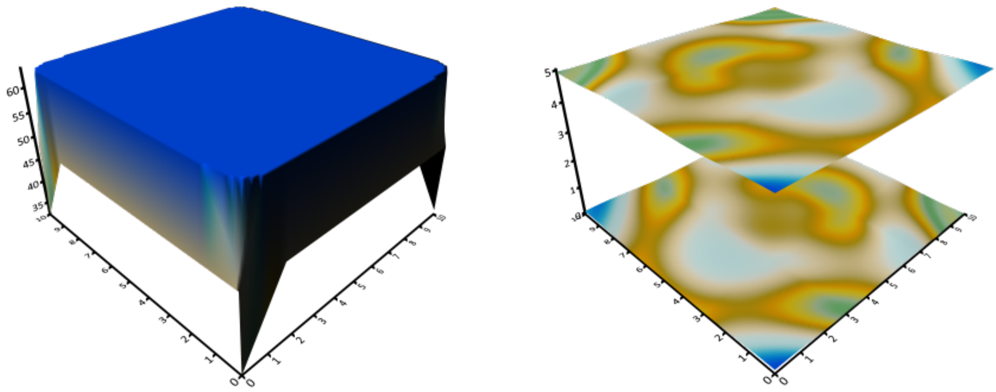


(**i**) Polynomial Regression (**j**) Modified Shepard's Method



(**k**) Data Metrics (**l**) Moving Average

Figure 2. 3D surface plots of the trigonometric surfaces generated using twelve different grid-ding methods in Surfer.

3.2. Results and Interpretation of Polynomial Surfaces

The upper and lower Hypothetical surfaces in this group were also generated using Py-thon, as defined by equations (3) and (4), respectively. The analytical volumes between these surfaces were precisely determined using analytical integration, resulting in a value of 7160.98753613601 cubic units. Figure 3 depicts the shape of these surfaces as created with Python.

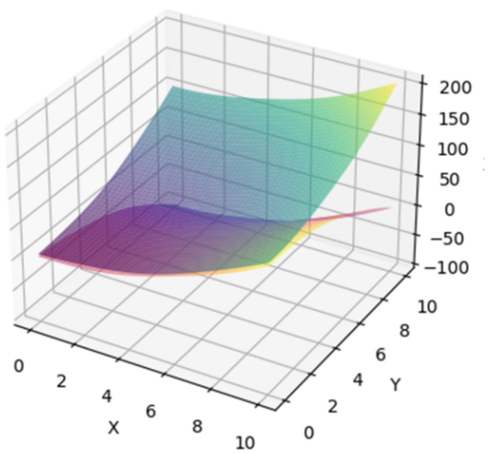


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Figure 3. Generated polynomial surfaces using Python, defined by equations (3) and (4), with an analytical volume of 7160.98753613601 cubic units.

To guarantee the precision and reliability of the volume computations, the surfaces were plotted using the Surfer software with twelve alternative gridding methods. The goal was to compare the numerical volumes generated using Surfer's gridding and numerical integration techniques to the analytically calculated volume.

After examining the volumes computed for the Polynomial Surfaces group using different gridding methods and numerical integration techniques, some trends and observations can be identified.

Some of the methods that can generate precise volume measurements are Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, Radial Basis Function, Polynomial Regression, and Modified Shepard's Method. These methods provide exceptional concurrence with the analytical volume of 7160.98753613601 cubic units. The Absolute Errors (AE), Squared Errors (SE), and Absolute Percentage Errors (APE) for these approaches are small or nil, indicating a high level of accuracy and precision across all three numerical integration techniques: Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule.

Nevertheless, the Natural Neighbor approach displays substantial deviations from the analytical volume. The computed volumes yield elevated AE, SE, and APE values, suggesting diminished trustworthiness for this dataset.

Remarkably, Polynomial Regression and Moving Average approaches yielded identical volume measurements despite apparent differences in their visual representations. Although the surface plots produced using these approaches did not visually correspond to the surfaces generated by Python, the volume computations utilizing numerical integration techniques were precise, resulting in a computed volume of 7160.98753613601 cubic units.

In contrast, the Data Metrics method exhibits inadequate performance, displaying significant inaccuracies across all integration techniques. The computed volumes deviate greatly, indicating that Data Metrics is not appropriate for this dataset.

To summarize, the majority of gridding approaches, with the exception of Natural Neighbor and Data Metrics, yield precise and dependable volume estimations when combined with robust numerical integration techniques. This study emphasizes the significance of choosing suitable gridding techniques to guarantee accurate volume calculations, particularly in applications that demand high precision, such as environmental assessments and resource estimation.

3.3. Results and Interpretation of Exponential Trigonometric Surfaces

Equations (5) and (6), which determine the upper and lower Hypothetical surfaces in the third group, were developed in Python. Using analytical integration, the analytical volumes between these surfaces were exactly determined, and the result was 1035.03930118278 cubic units. The shape of these surfaces as produced by Python is shown in Figure 4.

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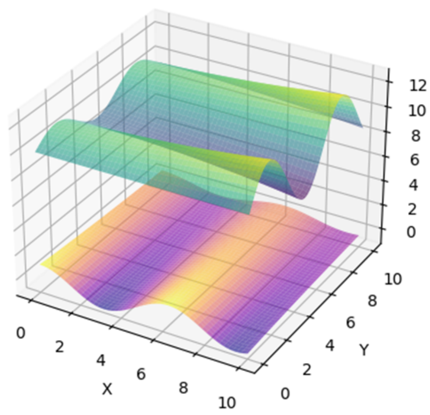

Figure 4. Generated exponential trigonometric surfaces using Python, defined by equations (5) and (6), with an analytical volume of 1035.03930118278 cubic units.

The surfaces were additionally plotted using the Surfer program, employing a total of twelve gridding techniques, in order to guarantee the precision and dependability of the volume computations. The objective was to compare the numerical volumes that were achieved with the analytically estimated volume using Surfer's gridding and numerical integration techniques.

When comparing the volumes computed using the twelve gridding methods with the analytically determined volume of 1035.03930118278 cubic units, several observations can be made about the precision and accuracy of these methods. The three numerical integration techniques used for this analysis are the Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule.

The following methods consistently produced accurate results across all numerical integration techniques: Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, Radial Basis Function, and Modified Shepard's Method. The analytical volume was closely followed by these approaches, which displayed minimal Absolute Percentage Errors (APE), Squared Errors (SE), and Absolute Errors (AE).

For example, Kriging proved to be very accurate, whereas the volumes computed using the Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule all showed very little inaccuracies. Volumes calculated using Inverse Distance to A Power were also spot on with the analytical value, proving the method's reliability.

However, there were notable differences between the results obtained using the Data Metrics and Natural Neighbor approaches. The increased AE, SE, and APE values are a reflection of the large analytical volume variations caused by Natural Neighbor. It appears that Natural Neighbor is not as trustworthy on this particular dataset. Data Metrics was clearly not made for this kind of data because it had terrible performance across all integration methods, displaying large failures.

Furthermore, the surface plots created by the Polynomial Regression and Moving Average methods failed to visually mimic the surfaces provided by the original Python code. There is a difference between visual and volumetric accuracy, since these methods produced precise volume estimates with little numerical errors, even if they didn't look similar.

All three types of surfaces—trigonometric, polynomial, and exponential trigonometric—had the same results. The reliability of some gridding methods is demonstrated by their constant performance across various surface types, while the importance of selecting appropriate

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gridding approaches based on specific topographical characteristics is highlighted by the observed disparities in other methods.

When the twelve surface plots created by Surfer were compared to the original Python surfaces in the three categories of trigonometric, polynomial, and exponential trigonometric surfaces, nine of the twelve plots were found to be an exact match. Gridding methods such as Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Natural Neighbor, Nearest Neighbor, Local Polynomial, Radial Basis Function, and Modified Shepard's Method are shown to be accurate by these consistent results.

But surfaces made using Data Metrics, Moving Average, and Polynomial Regression all have noticeable differences. The surface plots generated by these three approaches were drastically different from the ones provided by Python. It is worth mentioning that, even though they don't look similar, the Moving Average and Polynomial Regression approaches produced accurate volumes when numerically integrated. The need of thorough validation through numerical comparison and visual inspection is underscored by the fact that these gridding methods significantly differ in their visual and volumetric correctness.

In all three sets of surfaces, the same pattern of visual and volumetric matching was accurate with the same 9 approaches, whereas the same 3 methods constantly showed disparities. Based on the findings that have been replicated, it appears that the three methods that yielded inconsistent results may be more affected by specific surface data properties that are underrepresented in the surfaces generated by Python. Considerations such as the unique mathematical characteristics of the gridding techniques employed and differences in data density at the local level could fall into this category.

4. DISCUSSION

The attained results warrant a division of the discussion into two main sections to ensure a thorough comprehension and interpretation. The first section of our analysis investigates whether the volumes of material, determined by three numerical integration methods in Surfer program, align precisely with the volumes estimated analytically using distinct equations in Python. In the second section, another inquiry has been raised: "To what extent do the surfaces in the three groups, which are produced using grid files generated by different gridding methods in Surfer software, resemble the surfaces generated from various equations in Python?"

The responses to these inquiries will offer valuable perspectives on the precision and reliability of the numerical integration methods and gridding methods employed in this investigation.

4.1. First Group: Trigonometric Surfaces

Upon closer examination of the data for the trigonometric surface, it is evident that five techniques (Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, and Modified Shepard's Method) demonstrated complete consistency in the volumes predicted using the three numerical integration methods compared to those acquired using analytical integration. These methods also attained a perfect correspondence between the surface shape created by Surfer software and the one plotted using Python. Thus, it can be concluded that if natural topographies resemble these hypothetical rugged terrains with mountain peaks or hills and low valleys, these five techniques are highly recommended for obtaining the most precise depiction of such surfaces and for calculating volumes between these surfaces.

The exceptional precision observed with Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, and Modified Shepard's Meth-

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od can be ascribed to several variables. These methods are recognized for their mathematical robustness, which ensures dependable and consistent outcomes across a range of applications (Belkhiri et al., 2017). Their main emphasis is on local interpolation, which guarantees that the computed values closely correspond to the actual data points, hence maintaining the inherent patterns and characteristics of the terrain (Shukla et al., 2019). Moreover, these techniques exhibit efficient smoothing capabilities, which are crucial for dealing with the noise and abnormalities frequently encountered in real-world data (Yanalak, 2003).

In addition, these techniques are flexible and capable of being adjusted to various data sets and geographical characteristics, rendering them appropriate for a diverse array of uses (Yang et al., 2004). The interpolation and volume calculation methods are based on solid mathematical concepts, which guarantees their reliability and accuracy. This is supported by well-established theoretical underpinnings (Schaum, 2008). By incorporating these techniques into calculations of surface area and volume, researchers and professionals can get a significant level of accuracy, which is essential for accurately depicting natural terrains and for making well-informed judgments based on these calculations.

Furthermore, there are three other methods that have remarkably reduced error margins in volume calculations when compared to analytically computed volumes and those obtained using numerical integration, as discussed before. Furthermore, the surfaces produced by all three techniques in Surfer program are completely congruent with those plotted using Python. The three methods used are Kriging, Radial Basis Function, and Local Polynomial.

Kriging, a geostatistical technique, is notable for its sophisticated interpolation skills that consider spatial autocorrelation, resulting in a precise representation of surface topography and accurate volume calculations. The approach employs variogram models to generate optimal linear unbiased predictions of intermediate values, successfully capturing the spatial structure and variability of the data.

Radial Basis Function (RBF) interpolation is well-known for its ability to smoothly and flexibly fit complicated surfaces. Radial basis function (RBF) approaches employ radial basis functions to interpolate the values of a function across a specified number of points, resulting in a seamless and precise representation of the surface. The method is highly efficient in handling dispersed data points and yields little error in volume computations due to its intrinsic smoothness and adaptability (Bronowicka-Mielniczuk, et al., 2019).

Local polynomial interpolation, which utilizes polynomial equations suited to specific regions, has exceptional precision. This approach demonstrates enhanced capability in accommodating fluctuations in data by employing a polynomial equation that is fitted within a specific vicinity of each data point. Consequently, it captures localized patterns and guarantees a very accurate representation of the actual topography in the interpolated surface. By employing local fitting, the influence of outliers is minimized, leading to accurate volume computations (Celant & Broniatowski, 2016).

Collectively, these techniques showcase their expertise in precisely depicting and computing volumes for surfaces that imitate natural landscapes with different levels of intricacy. The use of spatial autocorrelation analysis in Kriging, the inherent smoothness of Radial Basis Function interpolation, and the local flexibility of Local Polynomial interpolation collectively contribute to their exceptional accuracy and minimal margin of error.

Following the highly accurate methods, it has been found that the Natural Neighbor interpolation approach has a significantly greater margin of error in volume computations compared to all other gridding methods.

The Trapezoidal Rule yields a numerical volume of 489.95000510152, which corresponds to a substantial Absolute Error (AE) of 10.04999489848, a Squared Error (SE) of

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101.002397459474, and an Absolute Percentage Error (APE) of 2.009998979696%, which indicates a substantial divergence from the analytical volume.

For Simpson's Rule, the computed volume is 491.61791880647, which, although significantly improved, still exhibits significant errors with an absolute error (AE) of 8.38208119353 and an absolute percent error (APE) of 1.67641623871%.

The Simpson's 3/8 Rule yields a numerical volume of 492.45293847567, indicating a modest improvement but still considerable inaccuracies with an absolute error (AE) of 7.54706152433 and an absolute percentage error (APE) of 1.50941230487%.

The significant inaccuracies reported in the Natural Neighbor approach can be due to various fundamental features of this interpolation technique. The process of Natural Neighbor interpolation involves creating natural neighbor regions around individual points and utilizing these regions to interpolate data. This approach is particularly responsive to the geographic arrangement and concentration of the input data points. Irregularly spaced data points or locations with sparse data can result in substantial errors when using interpolation (Conn et al., 2008). Despite the Natural Neighbor interpolation approach showing considerable inaccuracies in volume estimates, the forms of the resulting surfaces were completely consistent with those produced using Python. The comparatively elevated inaccuracies in Natural Neighbor interpolation emphasize the significance of choosing a suitable gridding technique that aligns with the characteristics and dispersion of the data.

Finally, it's critical to talk about the three techniques that were left out. The first method, Polynomial Regression, produced horizontal surfaces that were 100% different from the actual shape of the original surfaces generated by Python, despite achieving a perfect match with the volumes calculated through analytical integration with a 0% error rate. The nature of polynomial regression, which tends to smooth out the data and may cause complex surface properties to be oversimplified, is responsible for this notable discrepancy. The approach fails to capture the detailed topographical changes due to this smoothing effect, resulting in the generation of unrealistic horizontal surfaces rather than the rugged terrain features found in the original data.

Moving Average is the second of the three approaches that are not included. Although there were only a few minor inaccuracies in the volume calculations, the surfaces produced by the Moving Average approach differed dramatically from the initial surfaces developed using Python. These modified or smoothed surfaces can be referred to as adjusted surfaces. The mismatch occurs because the Moving Average approach naturally mitigates the data, diminishing the intricacy and intricateness of the topographical characteristics. Consequently, the surfaces produced fail to accurately depict the intricate topographical fluctuations, resulting in a loss of precision in reflecting the real terrain (Lee, 2019).

The Data Metrics approach is the third method that is excluded. Remarkably, the Data Metrics approach produces parallel surfaces that perfectly align with each other, leading to an estimated volume of zero between the surfaces. This suggests that the approach is unable to accurately record any differences in the terrain that occur in three dimensions. As a result, it cannot be trusted for accurately capturing 3D topographical surfaces or computing volumes between these surfaces. The method's inability to provide precise representations is attributed to its inherent constraints in dealing with intricate terrain characteristics and spatial variations, resulting in oversimplification and loss of critical details (Lee, 2019).

The alignment of surfaces made using Surfer software and those generated from equations using Python showed a high degree of consistency across all three surface groupings: Trigonometric surfaces, Polynomial surfaces, and Exponential Trigonometric surfaces. Therefore, the discussion regarding the alignment of surfaces generated using Surfer software with those generated from equations using Python will not be repeated when addressing the remaining two groups: Polynomial Surfaces and Exponential Trigonometric Surfaces. The discussion will focus on the outcomes relating to the consistency of volume computations among surfaces for each gridding technique, except the three approaches that were previously excluded in the first group, as they were also discarded in the subsequent two groups.

4.2. Second Group: Polynomial Surfaces

For the Polynomial Surfaces group, none of the three numerical integration approach using different gridding methods achieved a perfect match with the analytically computed volume. This can be attributed to many aspects associated with the characteristics of polynomial surfaces. Firstly, polynomial surfaces can exhibit small, intricate variations in slope and curvature. Some numerical integration methods may not accurately capture these small changes. In addition, several numerical integration methods exhibit sensitivity to minor alterations in the data, resulting in modest inconsistencies in the computed volume.

For polynomial surfaces, techniques such as Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, and Modified Shepard's Method demonstrated remarkable precision by effectively capturing the spatial variability in the data. These methods are highly proficient in effectively managing surfaces that exhibit gradual changes, a common trait of polynomial surfaces.

The Radial Basis Function approach exhibited exceptional performance, showcasing its resilience in accurately interpolating spatial data.

However, techniques such as Local Polynomial, Natural Neighbor, and Polynomial Regression exhibited greater inaccuracies. The Local Polynomial approach may add bias as it has a propensity to reduce variances, whilst the Natural Neighbor and Polynomial Regression methods may not adequately capture the intricate complexity of the polynomial surface, resulting in larger mistakes.

Overall, while dealing with polynomial surfaces, it is advisable to utilize techniques such as Kriging, Inverse Distance to A Power, and Triangulation with Linear Interpolation due to their high level of precision. Conversely, methods such as Moving Average and Data Metrics should be disregarded as they are not as accurate.

4.3. Third Group: Exponential Trigonometric Surfaces

The Exponential Trigonometric Surfaces, which have an analytical volume of 1035.03930118278 cubic units, yielded interesting results when compared to the Polynomial Surfaces and Trigonometric Surfaces in terms of the various gridding methods used.

The methods employed, including Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, and Modified Shepard's Method, demonstrated remarkable precision, especially when combined with Simpson's 3/8 Rule. The absolute errors were exceedingly minimal, nearly achieving a perfect match with the analytical volume. These approaches exhibited the lowest Absolute Error (AE), Squared Error (SE), and Absolute Percentage Error (APE), indicating their ability to effectively handle the spatial fluctuations inherent in exponential trigonometric surfaces. The consistent performance of these surfaces in varied contexts highlights their trustworthiness in diverse geospatial applications.

The Radial Basis Function approach demonstrated exceptional accuracy with minimal errors. This demonstrates its consistent and reliable performance in the Polynomial Surfaces category, hence affirming its strength and trustworthiness.

Conversely, the Local Polynomial and Polynomial Regression techniques exhibited greater errors. While their values are relatively low, they suggest a slight divergence from the analytical volume. These results indicate that Polynomial Regression may have problems in

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accurately representing exponential trigonometric surfaces. The results align with those seen in the Polynomial Surfaces group, where Polynomial Regression similarly exhibited noteworthy inaccuracies.

The Natural Neighbor technique displayed substantial inaccuracies. The Natural Neighbor approach exhibited a significant absolute error (AE) of 19.99433837328000 when used to the Trapezoidal Rule, and similarly substantial mistakes were observed with other numerical integration methods. These results suggest that the method struggles to accurately handle the intricate nature of exponential trigonometric surfaces. Similarly, the Polynomial Surfaces group found that Natural Neighbor exhibited substantial inaccuracies, thereby validating its limitations in adequately representing intricate surface geometries.

Ultimately, while dealing with exponential trigonometric surfaces, it is advisable to utilize techniques such as Kriging, Inverse Distance to A Power, and Triangulation with Linear Interpolation due to their remarkable precision, which is comparable to their effectiveness in Polynomial Surfaces.

Table 2 provides a thorough categorization of several gridding methods, taking into account their precision and appropriateness for generating surfaces and calculating volumes between those surfaces. The performance of each method was assessed by comparing the numerical integration results obtained from the Surfer software, with 12 distinct gridding methods, with the volumes estimated analytically using Python scripts. In addition, the surfaces produced using Python were compared to those plotted using various gridding methods in Surfer.

The categorization of the consistency between volumes computed through numerical integration using various gridding methods in Surfer and volumes computed using analytical integration in Python is organized into five ranks, each denoted by a capital letter. If the volume value is Perfect consistent, it is designated as the letter A. If it is remarkably consistent, it is assigned the letter B. If it demonstrates consistency with only a slight inaccuracy, it is assigned the grade of C. If it is inconsistent with a significant error compared to other methods, it is designated with the letter D. If it is completely inconsistent, it is designated with the letter E.

Table 2. Categorization of Gridding Methods According to Accuracy and Applicability for Surface Representation and Volume Calculation.

Gridding Method	Group I: Trigonometric Surfaces		Group II: Polynomial Surfaces		Group III: Exponential Trigonometric Surfaces	
	Accuracy					
	Volume Calculation	Surface Representation	Volume Calculation	Surface Representation	Volume Calculation	Surface Representation
1*	B	matched	B	matched	B	matched
2	A	matched	B	matched	B	matched
3	A	matched	B	matched	B	matched
4	A	matched	B	matched	B	matched
5	D	matched	D	matched	D	matched
6	A	matched	B	matched	B	matched
7	B	matched	D	matched	C	matched
8	B	matched	B	matched	B	matched
9	A	Not matched	D	Not matched	D	Not matched
10	A	matched	B	matched	B	matched
11	E	Not matched	E	Not matched	E	Not matched

12	B	Not matched	D	Not matched	D	Not matched

* 1= Kriging, 2= Inverse Distance to A Power, 3= Triangulation with Linear Interpolation, 4 =Minimum Curvature, 5= Natural Neighbor, 6= Nearest Neighbor, 7= Local Polynomial, 8= Radial Basis Function, 9= Polynomial Regression, 10= Modified Shepard's Method, 11=Data Metrics, and 12= Moving Average

Table 2 briefly presented the findings of the investigation in a manner that facilitates rapid understanding. The comprehensive overview will be provided in the subsequent section, "Conclusion."

5. CONCLUSION

This study assessed the precision and reliability of different gridding methods in creating surfaces and determining volumes in three separate types of terrains: Trigonometric Surfaces, Polynomial Surfaces, and Exponential Trigonometric Surfaces. The selection of these surfaces was based on their ability to accurately depict various topographical characteristics commonly observed in natural environments.

The Trigonometric Surfaces were selected to mimic rough, mountainous terrains characterized by conspicuous peaks and deep valleys. The Polynomial Surfaces were chosen to replicate gradually sloping mountainous areas devoid of prominent peaks or valleys. Finally, the Exponential Trigonometric Surfaces were created to depict high mountains interspersed with low-lying valleys. Every type of surface presented distinct difficulties for generating the surface and calculating its volume, resulting in a thorough evaluation of the effectiveness of the gridding approaches.

The results showed that there are seven methods that demonstrated either perfect or remarkable consistency throughout the three surface groups, in terms of both volume computations and the representation of the created surfaces. These methods are: Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, Radial Basis Function, and Modified Shepard's Method. These approaches demonstrated minimum absolute errors and great precision for all categories of surfaces, giving them reliable options for geospatial applications that involve intricate terrains.

Natural Neighbor method was found to be inconsistent across all three surface groups and was assigned the letter D, despite the fact that the surfaces generated using different gridding methods matched those generated by Python. It had a tendency to aggressively smooth down data, which resulted in distorting the true surface forms and introducing errors.

Local Polynomial Method produced varying results over the three surface groups. In the first group it was remarkably consistent; in the second group it was inconsistent; in the third group it was consistent. It may not be appropriate for terrains with substantial changes, as demonstrated in this study.

Polynomial Regression and Moving Average are both wholly rejected and excluded from the current study for the purpose of representing surfaces. This is due to the fact that the surfaces they generated were not matched with the precise surfaces that were generated using Python. The Moving Average approach produced smoothed or improved surfaces that deviated from the original topography.

Finally, the Data Metrics method is completely dismissed and omitted from the pre-sent study for its inability to accurately depict surfaces and compute volumes. This method not only fails to accurately match with the surfaces generated using Python, but it also causes the upper surface to completely overlap with the bottom surface, resulting in the materials between the surfaces being obscured or lost. As a result, the volumes obtained using this method are zero, rendering it entirely inappropriate for both calculating volumes and depicting three-dimensional topography.

In summary, it can be concluded that, for precise surface generation and volume calculation in geospatial applications, it is strongly advised to utilize methods such as Kriging, Inverse Distance to A Power, Triangulation with Linear Interpolation, Minimum Curvature, Nearest Neighbor, Radial Basis Function, and Modified Shepard's Method. These methods are highly proficient at capturing the complexities of many terrains, ranging from rugged Mountains to gently sloping plains.

This study emphasizes the significance of choosing suitable gridding techniques according to the distinct features of the terrain in order to guarantee reliable and precise geospatial analysis. Researchers and practitioners can improve the accuracy of their topographical and volumetric investigations by comprehending the advantages and constraints of each method and making informed judgments accordingly.

6. ACKNOWLEDGMENTS

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